First-Order Differential Equations Calculus of a Single Variable X. Du

- Solving a separable differential equation $\frac{dy}{dx} = g(x)h(y)$
  - 1) Separate variables. 2) Integrate. 3) Solve for *y*.
  - Don't forget equilibrium or constant solution!
- **Exponential Growth and Decay**
  - $\frac{dy}{dx} = ky$ Solution: $y = Ce^{kx}$
  - Applications:
    - Newton's Law of Cooling
    - Radioactive Decay
    - Continuously compounded interest
    - Discharging a capacitor
- **Logistical Growth and Decay**
  - $\frac{dy}{dx} = ky\left(1 - \frac{y}{L}\right)$ Solution: $y = \frac{L}{1 + Ce^{-kx}}$
  - Applications: Population growth
- **Homogenous Differential Equations**
  - $f(tx, ty) = t^n f(x, y)$
  - $y = vx$ $dy = vdx + xdv$ (Use product rule)
- **First-Order Linear Differential Equations**
  - $\frac{dy}{dx} + P(x)y = Q(x)$
  - $u(x) = e^{\int P(x)dx}$ Integrating factor (product rule). Use to multiply.
  - $y = \frac{1}{u(x)} \int Q(x)u(x)dx$ Solution
  - Applications:
    - Fluid friction and air resistance
    - Electrical circuits
    - Capacitor and inductor problems
    - Diffusion
- **Bernoulli Differential Equations**
  - $\frac{dy}{dx} + P(x)y = Q(x)y^n$
  - Substitute: $z = y^{1-n}$. Then solve as a first-order linear differential equation
  - Applications:
    - Fluid dynamics
    - Leaking tank (using Bernoulli's principle)

Further notes:

- More advanced techniques of solving differential equations will not be discussed in this context.